5) Hi this is Swapnila! I am a Student Success Coach and will be explaining some practice problems from the MA005 Calculus course. Today we will review an assigned problem from Unit 1. In the “Homework Assessment” you were asked to solve this problem:

A small airplane at an altitude of 5000 feet is flying East at 300 feet per second (a bit over 200 miles per hour), and you are watching it with a small telescope as it passes directly overhead. (Fig. 21)

a) What is the slope of the telescope 5, 10 and 20 seconds after the plane passes overhead?

b) What is the slope of the telescope t seconds after the plane passes overhead?

c) After it passes overhead, is the slope of the telescope increasing, decreasing, or staying the same?

Step 1: First find the distance traveled by the airplane at the given time:

\[
\text{Distance} = \text{velocity} \cdot \text{time}
\]

i) After \( t = 5 \) seconds.

\[
\text{Distance} = 300 \cdot 5 = 1500 \text{ feet}
\]

ii) After \( t = 10 \) seconds.

\[
\text{Distance} = 300 \cdot 10 = 3000 \text{ feet}
\]

iii) After \( t = 20 \) seconds.

\[
\text{Distance} = 300 \cdot 20 = 6000 \text{ feet}
\]

Step 2: Denote the angle at intersection of direction of airplane and telescope as \( \theta \):
**Step 3**: Plug the values of opposite and adjacent sides in the slope formula:

Slope formula:

\[ m = \tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \]

**i) After** \( t = 5 \) **seconds:**

The distance traveled by the airplane is 1500 feet, which is the adjacent side, and altitude of airplane is 5000 feet, which is the opposite side, as shown in the figure below:

Plug these values in the formula:

\[ m = \tan \theta = \frac{5000}{1500} \]

Simplify:
\[ m = \frac{10}{3} \]

ii) After \( t = 10 \) seconds:

The distance traveled by the airplane is 3000 feet, which is the adjacent side, and altitude of airplane is 5000 feet, which is the opposite side, as shown in the figure below:

![Diagram](image)

\[ \text{Adjacent side} = 3000 \text{ feet} \]
\[ \text{Opposite side} = 5000 \text{ feet} \]

Plug these values in the formula:

\[ m = \tan \theta = \frac{5000}{3000} \]

Simplify:

\[ m = \frac{5}{3} \]

iii) After \( t = 20 \) seconds:

The distance traveled by the airplane is 6000 feet, which is the adjacent side, and altitude of airplane is 5000 feet, which is the opposite side, as shown in the figure below:
Plug these values in the formula:

\[ m = \tan \theta = \frac{5000}{6000} \]

Simplify:

\[ m = \frac{5}{6} \]

**Conclusion:**

By using slope formula,

i) The slope of the telescope after 5 seconds is \( m = \frac{10}{3} \)

ii) The slope of the telescope after 10 seconds is \( m = \frac{5}{3} \)

iii) The slope of the telescope after 20 seconds is \( m = \frac{5}{6} \)

**b)** To find the slope of the telescope \( t \) seconds after the plane passes overhead, use the slope formula. Our approach for this problem will be as follows:

1. First find the distance traveled by the airplane at the given time \( t \).
2. Denote the angle at intersection of direction of airplane and telescope as \( \theta \).
3. Plug the values of opposite and adjacent side in the slope formula.

**Step 1:** First find the distance traveled by the airplane at the given time:
Distance = velocity \cdot time

At t seconds,

Distance = 300 \cdot t = 300t \text{ feet}

**Step 2:** Denote the angle at intersection of direction of airplane and telescope as $\theta$:

![Diagram of direction of airplane and telescope with an angle $\theta$.]

At $t$ seconds:
The distance traveled by the airplane is $300t \text{ feet}$, which is the adjacent side, and the altitude of the airplane is 5000 feet, which is the opposite side, as shown in the figure below:

![Diagram showing adjacent side and opposite side with values.]

**Step 3:** Plug the values of opposite and adjacent side in the slope formula.

Slope formula:

$m = \tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$

At $t$ seconds:
The distance traveled by the airplane is $300t \text{ feet}$, which is the adjacent side, and the altitude of the airplane is 5000 feet, which is the opposite side, as shown in the figure below:

Plug these values in formula:
\[ m = \tan \theta = \frac{5000}{300t}, \text{ for } t > 0 \]

Simplify:
\[ m = \frac{50}{3t}, \text{ for } t > 0 \]

**Conclusion:**

By using slope formula, the slope of the telescope after \( t \) seconds is \( m = \frac{50}{3t}, t > 0 \).

(c) The approach for this part will be as follows:

1. Use the result from part b, and check how the slope is changing as \( t \) increases.

**Step 1:** Use the result from part b, and check how the slope is changing as \( t \) increases.

The slope of the telescope after \( t \) seconds is \( m = \frac{50}{3t}, t > 0 \).

Here, the numerator remains constant, and the denominator depends on \( t \). As \( t \) increases, the denominator increases, and so, the result of the slope decreases.

**Conclusion:**

After it passes overhead, the slope of the telescope is decreasing.

Please let me know if you have any question on this problem, or on this topic generally. I will be here in the forum for the next hour.